GODEL's Incompleteness Theorem

There is a gap between truth and proof.

The Liar Paradox
This statement is false.
If true… but it says its false; If false then the statement must be true… but the statement says its false…

Consistency in mathematics:

Prove mathematics is consistent and every true statement should be provable.

There are true statements in mathematics that cannot be proved within any mathematical system.

Axioms: a rule that you believe is true. Creates a mathematical system.
We want a set of axioms from which we can deduce all true statements.

Maybe we haven't gotten all the axioms and if we find a statement that can't be proved, then we can add that to our set of axioms and it will expand what we can prove within mathematics.
We are trying to prove that there is a set of axioms for which we can deduce all truths of mathematics.

Godel coding. Any statement about numbers has its own particular code number. That is, every statement you make about mathematics can be turned into a number.
Every mathematical statement will have a unique number associated with it.
Is the particular statement provable by the axioms? Any statement whose code number is divisible by the code axioms means that is is provable by the axioms. So essentially we are representing logic and mathematics with mathematics (sort of on itself, discussing math with math).
This statement cannot be proved from the axioms has a code number. So it can be made into an equation therefore either true or false.
Assume its false—this statement **is** provable by the axioms, but a provable statement must be true. So we conclude its true, but assumed it was false. CONTRADICTION.
We assume math is consistent, so we cant have contradictions.
Its not false means It must be TRUE. Mathematics is either TRUE or FALSE.
This statement cannot be proved from the axioms is true. So a statement of mathematics which is true but cannot be proved!

In summary:
What happens is that we have axioms. We say "this statement cannot be proved from the axioms" which results in the statement being true, but indeed not proved by the set of axioms. So we can then take that statement and turn it into an axiom—thereby expanding the system of mathematics. But this doesn't help because no matter how much you attempt to expand mathematics by adding axioms, you will always be missing something—*a statement that can't be*

proved within that system of axioms

https://www.youtube.com/watch?v=O4ndIDcDSGc